

Question 6

Points constrained to the sphere

$$x^2 + y^2 + z^2 = 49$$

undergo small changes near $(2, 3, 6)$.

Tasks

1. Derive the differential constraint relating dx, dy, dz .
2. Estimate dz when $dx = 0.03$ and $dy = -0.02$.
3. Explain geometrically why the first-order change satisfies this relation.

Question 6 – Solution

Strategy. Differentiate the constant constraint; its total differential must vanish along allowed infinitesimal changes.

Step 1: Constraint

$$2x \, dx + 2y \, dy + 2z \, dz = 0,$$

so, because $z = 6 \neq 0$,

$$\boxed{dz = -\frac{x \, dx + y \, dy}{z}}.$$

Step 2: Estimate

$$dz = -\frac{2(0.03) + 3(-0.02)}{6} = -\frac{0.06 - 0.06}{6} = \boxed{0}.$$

Step 3: Geometry The radius vector $\langle x, y, z \rangle$ is normal to the sphere. The relation

$$\langle x, y, z \rangle \cdot \langle dx, dy, dz \rangle = 0$$

says the first-order displacement is perpendicular to that normal, hence tangent to the sphere. Here the chosen horizontal change is already orthogonal to the radius projection, so no first-order vertical correction is needed.