

Question 5

A cone has volume $V(r, h) = \frac{1}{3}\pi r^2 h$. At $r = 5$ cm, $h = 12$ cm, suppose $dr = 0.04$ cm and $dh = -0.10$ cm.

Tasks

1. Find dV exactly in terms of π .
2. Determine which measurement change dominates the estimate.
3. Compute the relative differential dV/V in two ways.

Question 5 – Solution

Strategy. Compute component contributions, then confirm the relative formula by logarithmic structure.

Step 1: Differential

$$dV = \frac{2\pi r h}{3} dr + \frac{\pi r^2}{3} dh.$$

At the given dimensions,

$$dV = 40\pi(0.04) + \frac{25\pi}{3}(-0.10) = 1.6\pi - \frac{5}{6}\pi = \boxed{\frac{23\pi}{30} \text{ cm}^3}.$$

Step 2: Compare The radial contribution is 1.6π ; the height contribution is $-5\pi/6$. The radial contribution dominates, so the net predicted change is positive.

Step 3: Relative form

$$\frac{dV}{V} = 2\frac{dr}{r} + \frac{dh}{h} = 2\frac{0.04}{5} - \frac{0.10}{12} = \boxed{\frac{23}{3000}}.$$

Also $V = 100\pi$, and $(23\pi/30)/(100\pi) = 23/3000$, verifying the result.