

Question 4

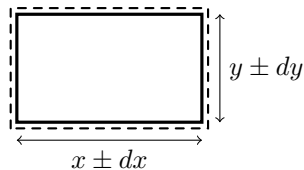
A rectangular plate has measured sides $x = 30.0$ cm and $y = 20.0$ cm, each accurate to within 0.1 cm. Its area is $A = xy$.

Tasks

1. Use differentials to estimate the maximum absolute area error.
2. Estimate the maximum relative and percentage errors.
3. Compare with the exact worst-case outward error.

Question 4 – Solution

Strategy. For a worst-case differential bound, replace signed increments by their maximum absolute values.



Step 1: Absolute error

$$dA = y \, dx + x \, dy.$$

Thus

$$|dA| \leq 20(0.1) + 30(0.1) = \boxed{5.0 \text{ cm}^2}.$$

Step 2: Relative error Since $A = 600 \text{ cm}^2$,

$$\frac{|dA|}{A} \approx \frac{5}{600} = \boxed{\frac{1}{120} \approx 0.00833},$$

or $\boxed{0.833\%}$.

Step 3: Exact comparison The largest outward area is

$$(30.1)(20.1) = 605.01,$$

so the exact positive error is 5.01 cm^2 . The differential misses only the product $dx \, dy = 0.01 \text{ cm}^2$.