

Question 3

For $f(x, y) = x^2 + y^2$, start at $(3, 4)$ and use $dx = 0.1$, $dy = -0.2$.

Tasks

1. Compute df .
2. Compute the exact change Δf .
3. Derive the discrepancy $\Delta f - df$ and explain its sign.

Question 3 – Solution

Strategy. Expand the exact finite change so the first-order and second-order parts separate visibly.

Step 1: Differential

$$df = 2x \, dx + 2y \, dy = 6(0.1) + 8(-0.2) = \boxed{-1.0}.$$

Step 2: Exact change

$$\Delta f = (3.1)^2 + (3.8)^2 - (3^2 + 4^2) = 9.61 + 14.44 - 25 = \boxed{-0.95}.$$

Step 3: Discrepancy Algebraically,

$$\Delta f = 2x \, dx + 2y \, dy + (dx)^2 + (dy)^2 = df + (dx)^2 + (dy)^2.$$

Therefore

$$\boxed{\Delta f - df = 0.01 + 0.04 = 0.05}.$$

It is nonnegative because the omitted terms are squares; the upward curvature makes the exact change exceed the linear prediction.