

Question 2

Use differentials to estimate

$$(4.02)^2\sqrt{8.97}.$$

Tasks

1. Choose a two-variable function and nearby convenient base point.
2. Compute the differential estimate.
3. State the estimated value and identify the neglected order of error.

Question 2 – Solution

Strategy. Let $f(x, y) = x^2\sqrt{y}$ and linearize its change from $(4, 9)$.

Step 1: Base data

$$f(4, 9) = 16(3) = 48, \quad dx = 0.02, \quad dy = -0.03.$$

$$f_x = 2x\sqrt{y}, \quad f_y = \frac{x^2}{2\sqrt{y}}.$$

At $(4, 9)$, $f_x = 24$ and $f_y = 8/3$.

Step 2: Differential

$$df = 24(0.02) + \frac{8}{3}(-0.03) = 0.48 - 0.08 = 0.40.$$

Step 3: Estimate

$$\boxed{(4.02)^2\sqrt{8.97} \approx 48 + 0.40 = 48.40}.$$

The differential keeps terms linear in dx, dy and neglects quadratic and higher-order increment effects.