

Question 1

For

$$z = x^2 e^y + \ln(x + y), \quad x + y > 0,$$

Tasks

1. Find the total differential dz .
2. Evaluate it at $(1, 1)$ for $dx = -0.02$, $dy = 0.03$.
3. Identify the contribution from each input change.

Question 1 – Solution

Strategy. Use $dz = f_x dx + f_y dy$ and evaluate coefficients before applying the increments.

Step 1: Partial derivatives

$$f_x = 2xe^y + \frac{1}{x+y}, \quad f_y = x^2e^y + \frac{1}{x+y}.$$

Thus

$$dz = \left(2xe^y + \frac{1}{x+y} \right) dx + \left(x^2e^y + \frac{1}{x+y} \right) dy.$$

Step 2: Evaluate At $(1, 1)$,

$$dz = (2e + \tfrac{1}{2})(-0.02) + (e + \tfrac{1}{2})(0.03).$$

Combining terms,

$$dz = -0.04e - 0.01 + 0.03e + 0.015 = \boxed{0.005 - 0.01e \approx -0.02218}.$$

Step 3: Contributions The x change contributes $-(0.04e + 0.01)$; the y change contributes $0.03e + 0.015$. Their signs oppose, and the x contribution has larger magnitude.