

Question 9

Suppose u satisfies $u_{tt} = 9u_{xx}$ and

$$u(x, t) = A \cos(2x) \cos(\omega t).$$

Tasks

1. Compute u_{tt} and u_{xx} .
2. Determine all real ω for nonzero A .
3. Verify the result and analyze the $A = 0$ edge case.

Question 9 – Solution

Strategy. Substitute the separated trial form and compare its common factors.

Step 1: Derivatives

$$u_{tt} = -A\omega^2 \cos(2x) \cos(\omega t), \quad u_{xx} = -4A \cos(2x) \cos(\omega t).$$

Step 2: Match For $A \neq 0$, equality for every (x, t) requires

$$-\omega^2 = 9(-4), \quad \omega^2 = 36,$$

so $\boxed{\omega = \pm 6}$.

Step 3: Verify Directly, $u_{tt} = -36u = 9(-4u) = 9u_{xx}$. Because $\cos(-6t) = \cos(6t)$, the signs produce the same function. If $A = 0$, the zero function satisfies the equation for every ω .