

Question 8

Let $f(x, y) = x^2y^2/(x^2 + y^2)$ away from $(0, 0)$ and $f(0, 0) = 0$.

Tasks

1. Find $f_x(0, y)$ and $f_y(x, 0)$ from definitions.
2. Compute $f_{xy}(0, 0)$ and $f_{yx}(0, 0)$.
3. Explain why axis data suffice here.

Question 8 – Solution

Strategy. Restrict first derivatives to an axis, then differentiate that restriction at zero.

Step 1: Axis first partials For $y \neq 0$,

$$f_x(0, y) = \lim_{h \rightarrow 0} \frac{h^2 y^2 / (h^2 + y^2)}{h} = \lim_{h \rightarrow 0} \frac{h y^2}{h^2 + y^2} = 0.$$

At $y = 0$ it is also zero. By symmetry, $f_y(x, 0) = 0$ for every x .

Step 2: Mixed values

$$f_{xy}(0, 0) = \lim_{k \rightarrow 0} \frac{f_x(0, k) - f_x(0, 0)}{k} = 0,$$

$$f_{yx}(0, 0) = \lim_{h \rightarrow 0} \frac{f_y(h, 0) - f_y(0, 0)}{h} = 0.$$

Thus $\boxed{f_{xy}(0, 0) = f_{yx}(0, 0) = 0}$.

Verification. Each definition uses only the relevant axis restriction, so off-axis values do not enter these two particular quotients.