

Question 7

For $w(x, y, z) = x^2yz^3 + \sin(xz)$:

Tasks

1. Find w_{xyz} .
2. Find w_{zzx} .
3. Evaluate both at $(1, 2, 0)$ and discuss derivative order.

Question 7 – Solution

Strategy. Follow the subscripts and keep polynomial and trigonometric contributions separate.

Step 1: w_{xyz}

$$w_x = 2xyz^3 + z \cos(xz), \quad w_{xy} = 2xz^3, \quad \boxed{w_{xyz} = 6xz^2}.$$

Step 2: w_{zzx}

$$\begin{aligned} w_z &= 3x^2yz^2 + x \cos(xz), \\ w_{zz} &= 6x^2yz - x^2 \sin(xz), \\ \boxed{w_{zzx} &= 12xyz - 2x \sin(xz) - x^2 z \cos(xz)}. \end{aligned}$$

Step 3: Evaluate Both happen to equal $\boxed{0}$ at $(1, 2, 0)$. Smoothness permits reordering derivatives that contain the same multiset of variables; xyz and zzx are different derivative types and need not agree generally.