

Question 4

Find the third-order derivative f_{xxy} for

$$f(x, y) = x^2 e^{xy}.$$

Tasks

1. Compute it in the encoded order.
2. Compute f_{yxx} independently.
3. Explain the agreement and evaluate at $(1, 0)$.

Question 4 – Solution

Strategy. Follow each subscript successively and retain all product-rule terms.

Step 1: Route x, x, y

$$\begin{aligned}f_x &= e^{xy}(2x + x^2y), \\f_{xx} &= e^{xy}(2 + 4xy + x^2y^2), \\f_{xxy} &= e^{xy}(6x + 6x^2y + x^3y^2).\end{aligned}$$

Step 2: Route y, x, x

$$\begin{aligned}f_y &= x^3e^{xy}, \quad f_{yx} = e^{xy}(3x^2 + x^3y), \\f_{yxx} &= e^{xy}(6x + 6x^2y + x^3y^2).\end{aligned}$$

Step 3: Verify Polynomial and exponential factors are smooth, so the derivative order may be permuted. At $(1, 0)$ both expressions equal $\boxed{6}$.