

**Question 3**

For  $u(x, y) = \ln(x^2 + y^2)$  on its natural domain:

**Tasks**

1. Find all second partial derivatives.
2. Show that  $u_{xx} + u_{yy} = 0$ .
3. Explain why the result cannot be extended to the origin by substitution.

**Question 3 – Solution**

**Strategy.** Differentiate the rational first partials and preserve the punctured-plane domain.

**Step 1: First partials**

$$u_x = \frac{2x}{x^2 + y^2}, \quad u_y = \frac{2y}{x^2 + y^2}.$$

**Step 2: Second partials**

$$\boxed{u_{xx} = \frac{2(y^2 - x^2)}{(x^2 + y^2)^2}}, \quad \boxed{u_{yy} = \frac{2(x^2 - y^2)}{(x^2 + y^2)^2}},$$
$$\boxed{u_{xy} = u_{yx} = -\frac{4xy}{(x^2 + y^2)^2}}.$$

**Step 3: Cancellation**

$$u_{xx} + u_{yy} = \frac{2(y^2 - x^2 + x^2 - y^2)}{(x^2 + y^2)^2} = \boxed{0}.$$

The origin is excluded from  $u$ , and every denominator above vanishes there; substitution cannot create a derivative at a point outside the domain.