

Question 2

Let $g(x, y) = e^{xy} \cos y$.

Tasks

1. Compute g_{xx} and g_{xy} .
2. Compute g_{yx} independently from g_y .
3. Evaluate them at $(0, \pi)$ and verify agreement.

Question 2 – Solution

Strategy. Keep product-rule terms grouped by their exponential factor.

Step 1: Differentiate from g_x Since $g_x = ye^{xy} \cos y$,

$$\boxed{g_{xx} = y^2 e^{xy} \cos y},$$

$$\boxed{g_{xy} = e^{xy} [(1 + xy) \cos y - y \sin y]}.$$

Step 2: Reverse order

$$g_y = e^{xy} (x \cos y - \sin y),$$

$$g_{yx} = ye^{xy} (x \cos y - \sin y) + e^{xy} \cos y = \boxed{e^{xy} [(1 + xy) \cos y - y \sin y]}.$$

Step 3: Evaluate At $(0, \pi)$, $e^{xy} = 1$, $\cos \pi = -1$, and $\sin \pi = 0$:

$$\boxed{g_{xx} = -\pi^2}, \quad \boxed{g_{xy} = g_{yx} = -1}.$$