

Question 1

For

$$f(x, y) = x^4y^2 - 3x^2y^3 + 2xy,$$

Tasks

1. Find $f_{xx}, f_{xy}, f_{yx}, f_{yy}$.
2. Evaluate them at $(1, -1)$.
3. Verify the mixed-partial equality algebraically.

Question 1 – Solution

Strategy. Compute both first partials before differentiating again.

Step 1: First partials

$$f_x = 4x^3y^2 - 6xy^3 + 2y, \quad f_y = 2x^4y - 9x^2y^2 + 2x.$$

Step 2: Second partials

$$\boxed{f_{xx} = 12x^2y^2 - 6y^3}, \quad \boxed{f_{xy} = 8x^3y - 18xy^2 + 2},$$
$$\boxed{f_{yx} = 8x^3y - 18xy^2 + 2}, \quad \boxed{f_{yy} = 2x^4 - 18x^2y}.$$

Step 3: Evaluate

$$\boxed{f_{xx}(1, -1) = 18}, \quad \boxed{f_{xy}(1, -1) = f_{yx}(1, -1) = -24}, \quad \boxed{f_{yy}(1, -1) = 20}.$$

The two mixed formulas coincide term by term, as expected for a polynomial.