

Question 10

Define

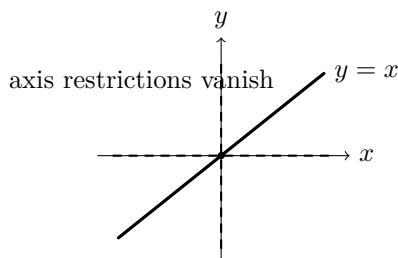
$$v(x, y) = \begin{cases} \frac{xy}{x^2 + y^2}, & (x, y) \neq (0, 0), \\ 0, & (x, y) = (0, 0). \end{cases}$$

Tasks

1. Show that both first partial derivatives exist at the origin.
2. Test continuity along $y = x$.
3. Explain what this example proves about partial derivatives and continuity.

Question 10 – Solution

Strategy. Partial derivatives inspect axis restrictions only; compare those restrictions with a diagonal approach.



Step 1: Axis difference quotients Since $v(h, 0) = 0$ and $v(0, k) = 0$,

$$v_x(0, 0) = \lim_{h \rightarrow 0} \frac{v(h, 0) - v(0, 0)}{h} = 0,$$

$$v_y(0, 0) = \lim_{k \rightarrow 0} \frac{v(0, k) - v(0, 0)}{k} = 0.$$

Thus

$$\boxed{v_x(0, 0) = v_y(0, 0) = 0}.$$

Step 2: Diagonal path Along $y = x$ with $x \neq 0$,

$$v(x, x) = \frac{x^2}{2x^2} = \frac{1}{2}.$$

Therefore $\lim_{x \rightarrow 0} v(x, x) = 1/2 \neq v(0, 0) = 0$.

Conclusion. The function is not continuous at the origin even though both first partial derivatives exist there. Hence

$$\boxed{\text{existence of } v_x, v_y \text{ at a point does not imply continuity there.}}$$