

Question 8

Determine whether constants a , b , and c exist such that

$$q(x, y) = ax^2 + bxy + cy^2$$

satisfies

$$q_x(1, 1) = 7, \quad q_y(1, 1) = 8, \quad q(1, 1) = 6.$$

Tasks

1. Translate the data into a linear system.
2. Solve for a, b, c or prove inconsistency.
3. Verify your conclusion against all three conditions.

Question 8 – Solution

Strategy. Compute the partial derivatives symbolically, evaluate at $(1, 1)$, and solve the resulting three equations.

Step 1: Equations

$$q_x = 2ax + by, \quad q_y = bx + 2cy.$$

Thus

$$2a + b = 7, \quad b + 2c = 8, \quad a + b + c = 6.$$

Step 2: Solve From the first two equations,

$$a = \frac{7-b}{2}, \quad c = \frac{8-b}{2}.$$

Substitution into $a + b + c = 6$ gives

$$\frac{7-b}{2} + b + \frac{8-b}{2} = \frac{15}{2},$$

which cannot equal 6. Therefore the supplied data are inconsistent.

Conclusion.

No constants a, b, c satisfy all three conditions.

Verification. In fact, adding the two derivative equations gives $2(a + b + c) = 15$, forcing $q(1, 1) = 15/2$, not 6.