

Question 7

For the surface

$$z = f(x, y) = x^2 + xy + y^2,$$

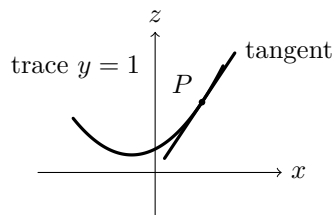
hold $y = 1$ fixed and examine the point $P = (1, 1, 3)$.

Tasks

1. Find $f_x(1, 1)$.
2. Obtain the tangent line to the trace $y = 1$ at P .
3. Verify the slope by differentiating the trace directly.

Question 7 – Solution

Strategy. Restrict the surface to the vertical plane $y = 1$; the resulting one-variable curve has slope $f_x(1, 1)$.



Step 1: Partial derivative

$$f_x = 2x + y, \quad \boxed{f_x(1, 1) = 3}.$$

Step 2: Trace and tangent At $y = 1$,

$$z = x^2 + x + 1.$$

The tangent line in the plane $y = 1$ passes through $(x, z) = (1, 3)$ with slope 3:

$$\boxed{y = 1, \quad z - 3 = 3(x - 1)}.$$

Step 3: Direct check Differentiating the trace gives $dz/dx = 2x + 1$, which equals 3 at $x = 1$, matching the partial derivative.