

Question 6

The equation

$$x^2 + y^2 + z^2 + xyz = 4$$

defines z locally as a function of x and y near $P = (1, 1, 1)$.

Tasks

1. Find z_x and z_y by partial implicit differentiation.
2. Evaluate them at P .
3. State the nonzero factor that permits solving for the derivatives.

Question 6 – Solution

Strategy. Differentiate the equation while treating $z = z(x, y)$, then collect the terms containing the requested derivative.

Step 1: Differentiate in x

$$2x + 2zz_x + yz + xyz_x = 0,$$

so

$$z_x = -\frac{2x + yz}{2z + xy}.$$

Step 2: Differentiate in y

$$2y + 2zz_y + xz + xyz_y = 0,$$

so

$$z_y = -\frac{2y + xz}{2z + xy}.$$

Step 3: Evaluate At $(1, 1, 1)$, the common denominator is $2z + xy = 3 \neq 0$. Hence

$$z_x(1, 1) = -1, \quad z_y(1, 1) = -1.$$

Substitution into the differentiated equations gives $2 + 2(-1) + 1 + (-1) = 0$ in each case.