

Question 5

Let

$$F(x, y, z) = x^2yz + \ln(x + z) - ye^z, \quad x + z > 0.$$

Tasks

1. Compute F_x , F_y , and F_z .
2. Evaluate all three at $(1, 2, 0)$.
3. Check which terms are constant in each differentiation.

Question 5 – Solution

Strategy. Differentiate with one variable active at a time and keep the logarithm's domain restriction.

Step 1: Partial derivatives

$$F_x = 2xyz + \frac{1}{x+z},$$

$$F_y = x^2z - e^z,$$

$$F_z = x^2y + \frac{1}{x+z} - ye^z.$$

Step 2: Evaluate At $(1, 2, 0)$,

$$F_x = 0 + 1 = \boxed{1}, \quad F_y = 0 - 1 = \boxed{-1},$$

$$F_z = 2 + 1 - 2 = \boxed{1}.$$

Verification. In F_x , $-ye^z$ is constant; in F_y , $\ln(x+z)$ is constant; in F_z , every term varies with z . The point satisfies $x+z=1>0$.