

Question 4

On the domain $x > 0$, define

$$p(x, y) = x^y.$$

Tasks

1. Find p_x by treating y as constant.
2. Find p_y using an exponential representation.
3. Evaluate both partial derivatives at $(e, 2)$ and verify their forms.

Question 4 – Solution

Strategy. Use the ordinary power rule for x and write $x^y = e^{y \ln x}$ when differentiating with respect to the exponent.

Step 1: x -partial With y fixed,

$$\boxed{p_x(x, y) = yx^{y-1}}.$$

Step 2: y -partial Since $p = e^{y \ln x}$ and $\ln x$ is constant with respect to y ,

$$\boxed{p_y(x, y) = x^y \ln x}.$$

Step 3: Evaluate

$$p_x(e, 2) = 2e, \quad p_y(e, 2) = e^2 \ln e = e^2.$$

Thus

$$\boxed{p_x(e, 2) = 2e, \quad p_y(e, 2) = e^2}.$$

Verification. For fixed $y = 2$, $p = x^2$ has derivative $2x$. For fixed $x = e$, $p = e^y$ has derivative e^y , confirming both values.