

Question 3

Let

$$h(x, y) = e^{xy} \sin(x + y).$$

Tasks

1. Compute h_x and h_y .
2. Evaluate both at $(0, \pi/2)$.
3. Identify the product-rule contribution that vanishes at the point.

Question 3 – Solution

Strategy. Use the product rule, differentiating each composite factor with only one variable active.

Step 1: x -partial

$$h_x = ye^{xy} \sin(x+y) + e^{xy} \cos(x+y) = e^{xy} [y \sin(x+y) + \cos(x+y)].$$

Step 2: y -partial

$$h_y = xe^{xy} \sin(x+y) + e^{xy} \cos(x+y) = e^{xy} [x \sin(x+y) + \cos(x+y)].$$

Step 3: Evaluate At $(0, \pi/2)$, $e^{xy} = 1$, $\sin(x+y) = 1$, and $\cos(x+y) = 0$. Therefore

$$\boxed{h_x(0, \pi/2) = \pi/2}, \quad \boxed{h_y(0, \pi/2) = 0}.$$

The cosine contribution vanishes in both partials; the $xe^{xy} \sin(x+y)$ contribution in h_y also vanishes because $x = 0$.