

Question 1

Let

$$f(x, y) = 4x^3y^2 - 5x^2y + 7y^3 - 6.$$

Tasks

1. Find f_x and f_y .
2. Evaluate both at $(1, -2)$.
3. Verify that each differentiation treats the other variable as constant.

Question 1 – Solution

Strategy. Differentiate term by term, holding the non-differentiated variable fixed.

Step 1: Differentiate with respect to x

$$f_x = 12x^2y^2 - 10xy.$$

The terms $7y^3$ and -6 are constant with respect to x .

Step 2: Differentiate with respect to y

$$f_y = 8x^3y - 5x^2 + 21y^2.$$

Step 3: Evaluate

$$f_x(1, -2) = 12(1)(4) - 10(1)(-2) = \boxed{68},$$

$$f_y(1, -2) = 8(1)(-2) - 5(1) + 21(4) = \boxed{63}.$$

Verification. The x -derivative lowers only powers of x ; the y -derivative lowers only powers of y . Substitution gives $48 + 20 = 68$ and $-16 - 5 + 84 = 63$.