

Question 9

Suppose (a, b) lies strictly inside the disk $x^2 + y^2 < 25$. Prove

$$\lim_{(x,y) \rightarrow (a,b)} \sqrt{25 - x^2 - y^2} = \sqrt{25 - a^2 - b^2},$$

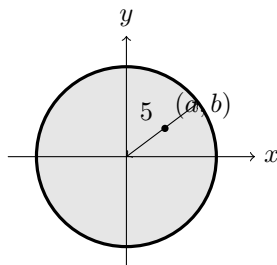
and give a concrete neighborhood that remains inside the domain.

Tasks

1. Find a positive safety margin to the boundary.
2. Choose a disk around (a, b) contained in the domain.
3. Justify the limit by composition.

Question 9 – Solution

Strategy. Use the triangle inequality to keep nearby points inside the radius-5 disk, then apply continuity.



Step 1: Margin Let $s = \sqrt{a^2 + b^2} < 5$. The radial margin is $5 - s > 0$.

Step 2: Neighborhood Choose

$$\delta_0 = \frac{5 - s}{2}.$$

If $\sqrt{(x - a)^2 + (y - b)^2} < \delta_0$, then by the triangle inequality

$$\sqrt{x^2 + y^2} \leq s + \sqrt{(x - a)^2 + (y - b)^2} < s + \frac{5 - s}{2} < 5.$$

Thus this entire neighborhood lies in the square-root domain.

Step 3: Composition The polynomial $25 - x^2 - y^2$ is continuous and remains positive there; $\sqrt{\cdot}$ is continuous on positive inputs. Their composition is continuous at (a, b) , proving the boxed limit stated in the question.