

Question 8

Prove directly from the ε - δ definition that

$$\lim_{(x,y) \rightarrow (1,-2)} (3x - 4y) = 11.$$

Tasks

1. Rewrite the output error.
2. Bound it by the Euclidean input distance.
3. Give an explicit valid choice of δ .

Question 8 – Solution

Strategy. Use Cauchy–Schwarz to compare the linear output change with the distance to $(1, -2)$.

Step 1: Error Let $h = x - 1$ and $k = y + 2$. Then

$$|(3x - 4y) - 11| = |3h - 4k|.$$

Step 2: Cauchy–Schwarz

$$|3h - 4k| \leq \sqrt{3^2 + (-4)^2} \sqrt{h^2 + k^2} = 5\sqrt{(x - 1)^2 + (y + 2)^2}.$$

Step 3: Choose δ Given $\varepsilon > 0$, set

$$\boxed{\delta = \varepsilon/5}.$$

If $0 < \sqrt{(x - 1)^2 + (y + 2)^2} < \delta$, then

$$|(3x - 4y) - 11| < 5\delta = \varepsilon.$$

This is exactly the two-variable limit definition, so the claimed limit is proved.