

Question 6

Let

$$D = \{(x, y) : x \geq 0, 0 \leq y \leq \sqrt{3}x\}, \quad f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}.$$

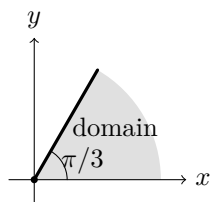
Find the limit of f as $(x, y) \rightarrow (0, 0)$ within D , or prove it does not exist.

Tasks

1. Describe the allowed directions.
2. Compare two valid boundary paths.
3. State the relative-limit conclusion.

Question 6 – Solution

Strategy. A relative limit uses only paths in the domain, but all such paths must still agree.



Step 1: Directions The wedge is $0 \leq \theta \leq \pi/3$. In polar form,

$$f = \frac{r^2 \cos^2 \theta - r^2 \sin^2 \theta}{r^2} = \cos 2\theta.$$

Step 2: Boundary paths Along $y = 0$ ($\theta = 0$), $f = 1$. Along $y = \sqrt{3}x$ ($\theta = \pi/3$),

$$f = \cos(2\pi/3) = -\frac{1}{2}.$$

Both paths stay in D and approach the origin.

Conclusion. Because the two permitted approaches disagree,

$$\lim_{\substack{(x,y) \rightarrow (0,0) \\ (x,y) \in D}} f(x,y) \text{ does not exist.}$$