

Question 3

Determine whether

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^4 + y^2}$$

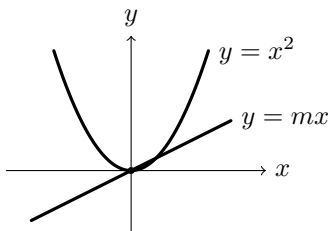
exists.

Tasks

1. Test every straight-line path $y = mx$.
2. Test a curved path suggested by the denominator.
3. Explain why the first test alone cannot prove existence.

Question 3 – Solution

Strategy. Compare straight lines with the balancing path $y = kx^2$, where x^4 and y^2 have the same order.



Step 1: Lines Along $y = mx$,

$$\frac{x^2(mx)}{x^4 + m^2x^2} = \frac{mx}{x^2 + m^2} \rightarrow 0$$

for $m \neq 0$; the path $y = 0$ also gives 0.

Step 2: Parabolas Along $y = kx^2$,

$$\frac{x^2(kx^2)}{x^4 + k^2x^4} = \frac{k}{1 + k^2}.$$

For example, $k = 1$ gives $1/2$, whereas $k = 0$ gives 0.

Conclusion. Since two paths produce different values,

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2y}{x^4 + y^2} \text{ does not exist.}$$

Agreement along all lines is necessary but not sufficient because nonlinearly approaching paths remain possible.