

Question 2

Evaluate

$$\lim_{(x,y) \rightarrow (1,2)} \frac{x^2 + xy - 3x - y + 2}{x - 1}.$$

Tasks

1. Explain why direct substitution is inconclusive.
2. Factor and simplify on the punctured domain.
3. Evaluate the limit and distinguish it from the function value.

Question 2 – Solution

Strategy. Factor the numerator to expose a common factor with the vanishing denominator.

Step 1: Indeterminate form At $(1, 2)$, numerator and denominator are both 0, so substitution gives $0/0$, not a limit value.

Step 2: Factor Treat the numerator as a polynomial in y :

$$x^2 - 3x + 2 + y(x - 1) = (x - 1)(x - 2) + y(x - 1) = (x - 1)(x + y - 2).$$

Whenever $x \neq 1$,

$$\frac{(x - 1)(x + y - 2)}{x - 1} = x + y - 2.$$

Step 3: Limit The simplified function is continuous, so

$$\lim_{(x,y) \rightarrow (1,2)} \frac{x^2 + xy - 3x - y + 2}{x - 1} = 1.$$

The original quotient is undefined on the entire line $x = 1$, including the target, but points of its domain approach $(1, 2)$ and the missing line does not change the limit.