

Question 9

A wire is wound exactly N times around a cylinder of radius R , rising uniformly through total height H . Model its centerline by

$$\mathbf{r}(t) = \left\langle R \cos t, R \sin t, \frac{H}{2\pi N} t \right\rangle, \quad 0 \leq t \leq 2\pi N.$$

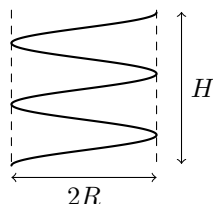
Tasks

1. Derive a formula for the wire length in terms of R, H, N .
2. For $R = 3$, $H = 24$, and length 30π , determine $N > 0$.
3. Verify whether the resulting value represents a whole number of turns.

Question 9 – Solution

Strategy. The model has constant speed; after integration, solve the design equation by squaring positive quantities.

wire wound around a cylinder



Step 1: General formula

$$\mathbf{r}'(t) = \left\langle -R \sin t, R \cos t, \frac{H}{2\pi N} \right\rangle,$$

so

$$\|\mathbf{r}'(t)\| = \sqrt{R^2 + \left(\frac{H}{2\pi N}\right)^2}.$$

Multiplying this constant speed by the parameter length $2\pi N$ gives

$$L = \sqrt{(2\pi NR)^2 + H^2}.$$

Step 2: Apply the data With $R = 3$, $H = 24$, and $L = 30\pi$,

$$(30\pi)^2 = (6\pi N)^2 + 24^2.$$

Therefore

$$\begin{aligned} 36\pi^2 N^2 &= 900\pi^2 - 576, \\ N^2 &= 25 - \frac{16}{\pi^2}. \end{aligned}$$

Since $N > 0$,

$$N = \sqrt{25 - \frac{16}{\pi^2}}.$$

Step 3: Feasibility Numerically, $N \approx 4.835$, which is not an integer. Thus the given continuous model has a positive solution, but the specifications are incompatible with a whole number of complete turns. This is a design-consistency conclusion, not an algebraic failure.