

Question 6

A path travels from $A = (-2, 0, 0)$ to $C = (0, 2, 0)$ and then to $B = (2, 0, 0)$ according to

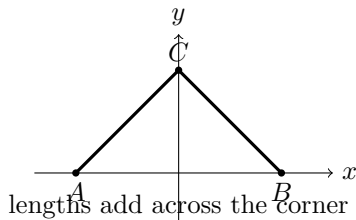
$$\mathbf{r}(t) = \begin{cases} \langle t, t+2, 0 \rangle, & -2 \leq t \leq 0, \\ \langle t, 2-t, 0 \rangle, & 0 < t \leq 2. \end{cases}$$

Tasks

1. Verify continuity at the joining parameter.
2. Compute the total arc length by splitting the integral.
3. Compare it with $\|B - A\|$ and explain the difference.

Question 6 – Solution

Strategy. Arc length is additive across a corner, even though the derivative changes abruptly there.



Step 1: Join check Both formulas give

$$\langle 0, 2, 0 \rangle$$

at $t = 0$ (the second by its right-hand limit), so the path is continuous at C .

Step 2: Length of each segment On the first branch,

$$\mathbf{r}'(t) = \langle 1, 1, 0 \rangle, \quad \|\mathbf{r}'(t)\| = \sqrt{2}.$$

On the second branch,

$$\mathbf{r}'(t) = \langle 1, -1, 0 \rangle, \quad \|\mathbf{r}'(t)\| = \sqrt{2}.$$

Therefore

$$\begin{aligned} L &= \int_{-2}^0 \sqrt{2} \, dt + \int_0^2 \sqrt{2} \, dt \\ &= 2\sqrt{2} + 2\sqrt{2} = \boxed{4\sqrt{2}}. \end{aligned}$$

Step 3: Chord comparison The endpoint distance is

$$\|B - A\| = \|\langle 4, 0, 0 \rangle\| = 4.$$

Since $4\sqrt{2} > 4$, the broken path is longer. Equality would require travel directly along the segment from A to B without the detour through C .