

Question 4

For

$$\mathbf{r}(t) = \langle 3t, 4t, 0 \rangle, \quad t \geq 1,$$

let $s(t)$ denote distance traveled from the starting parameter $t = 1$.

Tasks

1. Find $s(t)$ and its inverse $t(s)$.
2. Reparametrize the curve by arc length measured from $t = 1$.
3. Verify directly that the new parametrization has unit speed.

Question 4 – Solution

Strategy. Integrate speed from the specified base parameter, invert the resulting monotone function, and substitute.

Step 1: Arc-length function

$$\mathbf{r}'(t) = \langle 3, 4, 0 \rangle, \quad \|\mathbf{r}'(t)\| = 5.$$

Therefore

$$s(t) = \int_1^t 5 \, du = 5(t - 1).$$

Since $t \geq 1$, we have $s \geq 0$, and solving for t gives

$$\boxed{t(s) = 1 + \frac{s}{5}}.$$

Step 2: Substitute

$$\begin{aligned} \mathbf{R}(s) &= \mathbf{r}\left(1 + \frac{s}{5}\right) \\ &= \boxed{\left\langle 3 + \frac{3}{5}s, 4 + \frac{4}{5}s, 0 \right\rangle}, \quad s \geq 0. \end{aligned}$$

Step 3: Unit-speed verification

$$\mathbf{R}'(s) = \left\langle \frac{3}{5}, \frac{4}{5}, 0 \right\rangle,$$

so

$$\|\mathbf{R}'(s)\| = \sqrt{\frac{9}{25} + \frac{16}{25}} = 1.$$

Thus an increase of one unit in s corresponds to exactly one unit of distance along the curve.