

Question 2

A helix is parametrized by

$$\mathbf{r}(t) = \langle 5 \cos t, 5 \sin t, 12t \rangle, \quad 0 \leq t \leq 3\pi.$$

Tasks

1. Find its exact length.
2. Determine the length traveled during one complete revolution.
3. Explain why equal parameter intervals produce equal arc lengths here.

Question 2 – Solution

Strategy. Differentiate and look for constant speed.

Step 1: Speed

$$\mathbf{r}'(t) = \langle -5 \sin t, 5 \cos t, 12 \rangle.$$

Hence

$$\begin{aligned}\|\mathbf{r}'(t)\| &= \sqrt{25 \sin^2 t + 25 \cos^2 t + 144} \\ &= \sqrt{25 + 144} = 13.\end{aligned}$$

Step 2: Total length

$$L = \int_0^{3\pi} 13 \, dt = 13[t]_0^{3\pi} = \boxed{39\pi}.$$

Step 3: One revolution A complete revolution changes t by 2π . Therefore

$$L_{\text{rev}} = \int_t^{t+2\pi} 13 \, du = \boxed{26\pi}.$$

Because the speed is the same constant 13 at every parameter value, an interval of width Δt always contributes length $13\Delta t$. Thus equal parameter intervals have equal arc lengths.

Verification. The interval $[0, 3\pi]$ contains $3/2$ revolutions, and $(3/2)(26\pi) = 39\pi$, confirming the total.