

Question 3

Consider the plane curve

$$\mathbf{r}(t) = \langle t, t^2, 0 \rangle.$$

At $t = 1$, determine $\mathbf{T}$, $\mathbf{N}$, and $\mathbf{B}$.

Tasks

1. Differentiate and normalize to find $\mathbf{T}(1)$.
2. Differentiate the general $\mathbf{T}(t)$ before evaluating to find $\mathbf{N}(1)$.
3. Compute $\mathbf{B}(1)$ and interpret its direction.

Question 3 – Solution

Strategy. Computing $\mathbf{T}'(1)$ requires differentiating the normalized tangent, not merely normalizing $\mathbf{r}''(1)$.

Step 1: Unit tangent

$$\mathbf{r}'(t) = \langle 1, 2t, 0 \rangle, \quad \mathbf{T}(t) = \frac{\langle 1, 2t, 0 \rangle}{\sqrt{1 + 4t^2}}.$$

Hence

$$\boxed{\mathbf{T}(1) = \frac{1}{\sqrt{5}} \langle 1, 2, 0 \rangle}.$$

Step 2: Differentiate $\mathbf{T}$ Componentwise differentiation gives

$$\mathbf{T}'(t) = \frac{1}{(1 + 4t^2)^{3/2}} \langle -4t, 2, 0 \rangle.$$

At $t = 1$ this vector is a positive multiple of $\langle -2, 1, 0 \rangle$, whose length is $\sqrt{5}$. Therefore

$$\boxed{\mathbf{N}(1) = \frac{1}{\sqrt{5}} \langle -2, 1, 0 \rangle}.$$

Step 3: Binormal

$$\mathbf{B}(1) = \mathbf{T}(1) \times \mathbf{N}(1) = \frac{1}{5} \langle 1, 2, 0 \rangle \times \langle -2, 1, 0 \rangle = \boxed{\langle 0, 0, 1 \rangle}.$$

The curve lies in the xy -plane, so its binormal points perpendicular to that plane. The three vectors are unit length and mutually perpendicular.