

Question 2

For the circular helix

$$\mathbf{r}(t) = \langle 3 \cos t, 3 \sin t, 4t \rangle,$$

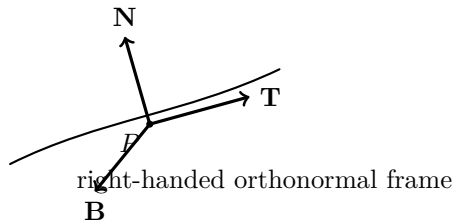
find the complete Frenet frame $\mathbf{T}(t)$, $\mathbf{N}(t)$, and $\mathbf{B}(t)$.

Tasks

1. Normalize $\mathbf{r}'(t)$.
2. Normalize $\mathbf{T}'(t)$.
3. Compute $\mathbf{B} = \mathbf{T} \times \mathbf{N}$ and verify orientation.

Question 2 – Solution

Strategy. The helix has constant speed, which makes each normalization especially transparent.

**Step 1: Unit tangent**

$$\mathbf{r}'(t) = \langle -3 \sin t, 3 \cos t, 4 \rangle, \quad \|\mathbf{r}'(t)\| = \sqrt{9 \sin^2 t + 9 \cos^2 t + 16} = 5.$$

Thus

$$\mathbf{T}(t) = \left\langle -\frac{3}{5} \sin t, \frac{3}{5} \cos t, \frac{4}{5} \right\rangle.$$

Step 2: Principal unit normal

$$\mathbf{T}'(t) = \left\langle -\frac{3}{5} \cos t, -\frac{3}{5} \sin t, 0 \right\rangle, \quad \|\mathbf{T}'(t)\| = \frac{3}{5}.$$

Therefore

$$\mathbf{N}(t) = \langle -\cos t, -\sin t, 0 \rangle.$$

Step 3: Binormal

$$\begin{aligned} \mathbf{B}(t) &= \mathbf{T}(t) \times \mathbf{N}(t) \\ &= \left\langle \frac{4}{5} \sin t, -\frac{4}{5} \cos t, \frac{3}{5} \right\rangle. \end{aligned}$$

Its norm is 1, and direct dot products give $\mathbf{T} \cdot \mathbf{N} = \mathbf{T} \cdot \mathbf{B} = \mathbf{N} \cdot \mathbf{B} = 0$. The defining cross product confirms the right-handed orientation.