

Question 1

For

$$\mathbf{r}(t) = \langle t^2, 2t, t^2 - 2t \rangle,$$

find the unit tangent vector at $t = 1$ and write the tangent line there.

Tasks

1. Compute the point and velocity vector.
2. Normalize the velocity to obtain $\mathbf{T}(1)$.
3. Write and verify a tangent-line parametrization.

Question 1 – Solution

Strategy. At a regular point, the unit tangent is $\mathbf{T} = \mathbf{r}' / \|\mathbf{r}'\|$.

Step 1: Point and velocity

$$\mathbf{r}(1) = \langle 1, 2, -1 \rangle, \quad \mathbf{r}'(t) = \langle 2t, 2, 2t - 2 \rangle,$$

so $\mathbf{r}'(1) = \langle 2, 2, 0 \rangle \neq \mathbf{0}$.

Step 2: Normalize

$$\|\mathbf{r}'(1)\| = \sqrt{2^2 + 2^2} = 2\sqrt{2}.$$

Therefore

$$\mathbf{T}(1) = \frac{\langle 2, 2, 0 \rangle}{2\sqrt{2}} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\rangle.$$

Step 3: Tangent line Either $\mathbf{r}'(1)$ or the parallel unit vector may be used:

$$\mathbf{L}(s) = \langle 1, 2, -1 \rangle + s \langle 1, 1, 0 \rangle.$$

At $s = 0$ the line passes through the curve point, and its direction $\langle 1, 1, 0 \rangle$ is one-half of $\mathbf{r}'(1)$.

Verification. The squared length of $\mathbf{T}(1)$ is $1/2 + 1/2 = 1$, as required.