

Question 4

Find the unique vector function $\mathbf{r}(t)$ satisfying

$$\mathbf{r}'(t) = \left\langle 2te^{t^2}, \frac{3}{1+t^2}, 4t^3 - 2t \right\rangle, \quad \mathbf{r}(0) = \langle 1, -2, 5 \rangle.$$

Tasks

1. Recover $\mathbf{r}(t)$ by componentwise antidifferentiation.
2. Use the initial vector to determine every constant.
3. Differentiate your final expression to verify it.

Question 4 – Solution

Strategy. Antidifferentiate each component and then use the initial vector to determine three constants.

Step 1: First component

$$\int 2te^{t^2} dt = e^{t^2} + C_1,$$

using the substitution $u = t^2$.

Step 2: Remaining components

$$\int \frac{3}{1+t^2} dt = 3 \arctan t + C_2,$$

and

$$\int (4t^3 - 2t) dt = t^4 - t^2 + C_3.$$

Thus

$$\mathbf{r}(t) = \left\langle e^{t^2} + C_1, 3 \arctan t + C_2, t^4 - t^2 + C_3 \right\rangle.$$

Step 3: Initial condition At $t = 0$,

$$\mathbf{r}(0) = \langle 1 + C_1, C_2, C_3 \rangle = \langle 1, -2, 5 \rangle.$$

Hence $C_1 = 0$, $C_2 = -2$, and $C_3 = 5$. Therefore

$$\boxed{\mathbf{r}(t) = \left\langle e^{t^2}, 3 \arctan t - 2, t^4 - t^2 + 5 \right\rangle}.$$

Verification. Componentwise differentiation gives $\left\langle 2te^{t^2}, 3/(1+t^2), 4t^3 - 2t \right\rangle$, and direct substitution gives the required initial vector. These three scalar initial-value problems each have only one integration constant, so the solution is unique.