

**Question 3**

Let

$$\mathbf{u}(t) = \langle t, t^2, 1 \rangle, \quad \mathbf{v}(t) = \langle \cos t, \sin t, t \rangle.$$

Without expanding first, compute at  $t = 0$ :

**Tasks**

1.  $\frac{d}{dt}(\mathbf{u} \cdot \mathbf{v})$ ,
2.  $\frac{d}{dt}(\mathbf{u} \times \mathbf{v})$ ,
3.  $\frac{d}{dt}[(t^2 + 1)\mathbf{v}(t)]$ .

State the differentiation rule used in each part.

**Question 3 – Solution**

**Strategy.** Apply the dot-product, cross-product, and scalar–vector product rules before inserting  $t = 0$ .

At  $t = 0$ ,

$$\mathbf{u} = \langle 0, 0, 1 \rangle, \quad \mathbf{u}' = \langle 1, 0, 0 \rangle, \quad \mathbf{v} = \langle 1, 0, 0 \rangle, \quad \mathbf{v}' = \langle 0, 1, 1 \rangle.$$

**Step 1: Dot product rule**

$$\begin{aligned} \frac{d}{dt}(\mathbf{u} \cdot \mathbf{v}) &= \mathbf{u}' \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{v}', \\ \frac{d}{dt}(\mathbf{u} \cdot \mathbf{v}) \Big|_0 &= \langle 1, 0, 0 \rangle \cdot \langle 1, 0, 0 \rangle + \langle 0, 0, 1 \rangle \cdot \langle 0, 1, 1 \rangle = \boxed{2}. \end{aligned}$$

**Step 2: Cross product rule** Order matters:

$$\begin{aligned} \frac{d}{dt}(\mathbf{u} \times \mathbf{v}) &= \mathbf{u}' \times \mathbf{v} + \mathbf{u} \times \mathbf{v}', \\ \frac{d}{dt}(\mathbf{u} \times \mathbf{v}) \Big|_0 &= \mathbf{0} + \langle 0, 0, 1 \rangle \times \langle 0, 1, 1 \rangle = \boxed{\langle -1, 0, 0 \rangle}. \end{aligned}$$

**Step 3: Scalar–vector rule** With  $f(t) = t^2 + 1$ ,

$$(f\mathbf{v})' = f'\mathbf{v} + f\mathbf{v}' \implies (f\mathbf{v})'(0) = 0\mathbf{v}(0) + 1\mathbf{v}'(0) = \boxed{\langle 0, 1, 1 \rangle}.$$