

Question 2

Let

$$\mathbf{r}(t) = \langle t^2 - 1, t^3 - 3t, 2t + 1 \rangle.$$

Tasks

1. Find $\mathbf{r}'(t)$ directly from the vector difference quotient.
2. Find the tangent line at the point corresponding to $t = 1$.
3. Verify that your direction vector is nonzero.

Question 2 – Solution

Strategy. Expand $\mathbf{r}(t+h) - \mathbf{r}(t)$ componentwise, divide by h , and then let $h \rightarrow 0$.

Step 1: Difference quotient

$$\begin{aligned}\frac{\mathbf{r}(t+h) - \mathbf{r}(t)}{h} &= \left\langle \frac{(t+h)^2 - t^2}{h}, \frac{(t+h)^3 - 3(t+h) - (t^3 - 3t)}{h}, \frac{2h}{h} \right\rangle \\ &= \langle 2t+h, 3t^2+3th+h^2-3, 2 \rangle.\end{aligned}$$

Taking $h \rightarrow 0$ yields

$$\boxed{\mathbf{r}'(t) = \langle 2t, 3t^2 - 3, 2 \rangle}.$$

Step 2: Point and direction At $t = 1$,

$$\mathbf{r}(1) = \langle 0, -2, 3 \rangle, \quad \mathbf{r}'(1) = \langle 2, 0, 2 \rangle.$$

The derivative is nonzero, so it supplies a valid tangent direction.

Step 3: Tangent line Using a new line parameter s ,

$$\boxed{\mathbf{L}(s) = \langle 0, -2, 3 \rangle + s \langle 2, 0, 2 \rangle}.$$

Substitution at $s = 0$ confirms the point, and the line direction is parallel to $\mathbf{r}'(1)$.