

Question 9

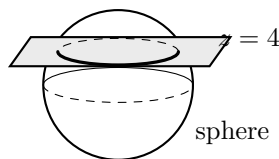
A curve is known to lie on the sphere $x^2 + y^2 + z^2 = 25$ and in the plane $z = 4$. Construct two vector parametrizations that traverse the entire curve exactly once in opposite directions. Choose both to start at $(3, 0, 4)$.

Tasks

1. Identify the intersection curve and its dimensions.
2. Construct the two oppositely oriented parametrizations.
3. Verify the surface constraints, starting point, and one-pass intervals.

Question 9 – Solution

Strategy. Substituting the fixed height into the sphere reveals a circle. The sign of the sine component controls orientation.



Step 1: Identify the circle With $z = 4$, the sphere equation becomes

$$x^2 + y^2 + 16 = 25 \implies x^2 + y^2 = 9.$$

The intersection is a radius-3 circle centered at $(0, 0, 4)$.

Step 2: First orientation A standard counterclockwise parametrization as viewed from the positive z -axis is

$$\mathbf{r}_+(t) = \langle 3 \cos t, 3 \sin t, 4 \rangle, \quad 0 \leq t < 2\pi.$$

It begins at $\mathbf{r}_+(0) = \langle 3, 0, 4 \rangle$ and initially enters the half-space $y > 0$.

Step 3: Opposite orientation Reverse the sign of the sine component:

$$\mathbf{r}_-(t) = \langle 3 \cos t, -3 \sin t, 4 \rangle, \quad 0 \leq t < 2\pi.$$

It has the same starting point but initially enters $y < 0$, so its orientation is opposite.

Verification. For either formula,

$$x^2 + y^2 + z^2 = 9(\cos^2 t + \sin^2 t) + 16 = 25.$$

The half-open interval covers every angular position once without duplicating the start at 2π .