

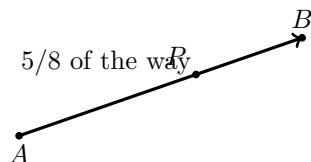
Question 8

Points $A = (-2, 1, 4)$ and $B = (6, -3, 0)$ are endpoints of a directed segment.

1. Construct a vector function $\mathbf{r}(t)$, $0 \leq t \leq 1$, that travels from A to B at a constant parameter rate.
2. Find the point reached when five-eighths of the parameter interval has elapsed.
3. Reparametrize the same directed segment on $-3 \leq u \leq 5$.

Question 8 – Solution

Strategy. A directed segment is an affine blend $(1 - t)A + tB$. A new interval requires a linear map back to $[0, 1]$.



Step 1: Segment formula First compute

$$B - A = \langle 8, -4, -4 \rangle.$$

Thus

$$\boxed{\mathbf{r}(t) = A + t(B - A) = \langle -2 + 8t, 1 - 4t, 4 - 4t \rangle}, \quad 0 \leq t \leq 1.$$

Indeed, $\mathbf{r}(0) = A$ and $\mathbf{r}(1) = B$.

Step 2: Division point At $t = 5/8$,

$$\mathbf{r}\left(\frac{5}{8}\right) = \left\langle -2 + 5, 1 - \frac{5}{2}, 4 - \frac{5}{2} \right\rangle = \boxed{\left\langle 3, -\frac{3}{2}, \frac{3}{2} \right\rangle}.$$

Step 3: New parameter interval The linear map taking $u = -3$ to $t = 0$ and $u = 5$ to $t = 1$ is

$$t = \frac{u + 3}{8}.$$

Substitution gives

$$\boxed{\mathbf{R}(u) = \left\langle u + 1, -\frac{1}{2}u - \frac{1}{2}, \frac{5}{2} - \frac{1}{2}u \right\rangle}, \quad -3 \leq u \leq 5.$$

Checking $u = -3$ and $u = 5$ recovers A and B in the required order.