

Question 7

Let

$$\mathbf{a}(t) = \langle t, 1 - t, 2t \rangle, \quad \mathbf{b}(t) = \langle 1, t, t^2 \rangle.$$

Tasks

1. Find the common domain of $\mathbf{a}/(t - 2)$ and $\mathbf{b}/(t + 1)$.
2. Compute $2\mathbf{a}(t) - \mathbf{b}(t)$.
3. Find all t for which $\mathbf{a}(t) \cdot \mathbf{b}(t) = 0$.

Question 7 – Solution

Strategy. Treat scalar division and vector algebra componentwise; for orthogonality, set the dot product equal to zero.

Step 1: Common domain The vector functions themselves are polynomial and defined for all real t . The scalar denominators impose

$$t - 2 \neq 0, \quad t + 1 \neq 0.$$

Therefore the common domain is

$$\boxed{\mathbb{R} \setminus \{-1, 2\}}.$$

Step 2: Vector combination

$$\begin{aligned} 2\mathbf{a}(t) - \mathbf{b}(t) &= \langle 2t, 2 - 2t, 4t \rangle - \langle 1, t, t^2 \rangle \\ &= \boxed{\langle 2t - 1, 2 - 3t, 4t - t^2 \rangle}. \end{aligned}$$

Step 3: Orthogonality

$$\begin{aligned} \mathbf{a}(t) \cdot \mathbf{b}(t) &= t(1) + (1 - t)t + (2t)t^2 \\ &= 2t - t^2 + 2t^3 \\ &= t(2t^2 - t + 2). \end{aligned}$$

The quadratic has discriminant $(-1)^2 - 4(2)(2) = -15 < 0$, so it has no real roots. Consequently

$$\boxed{t = 0}$$

is the only real solution. Directly, $\mathbf{a}(0) = \langle 0, 1, 0 \rangle$ and $\mathbf{b}(0) = \langle 1, 0, 0 \rangle$, whose dot product is zero.