

Question 5

Construct a vector parametrization of the curve where

$$x^2 + y^2 = 9 \quad \text{and} \quad z = x - 2y.$$

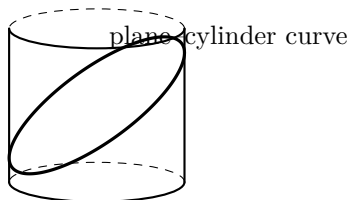
Choose the parametrization so that it begins at $(3, 0, 3)$ and initially moves toward points with $y > 0$. State a parameter interval that traces the curve exactly once.

Tasks

1. Parametrize the cylindrical constraint with the required starting direction.
2. Impose the plane equation to obtain the height component.
3. State and verify an interval that traces the curve exactly once.

Question 5 – Solution

Strategy. Parametrize the cylinder first; then use the plane equation to determine the remaining component.



Step 1: Parametrize the circle The cylinder equation suggests

$$x = 3 \cos t, \quad y = 3 \sin t,$$

because these expressions satisfy $x^2 + y^2 = 9$ identically.

Step 2: Impose the plane Substitute the chosen x and y into $z = x - 2y$:

$$z = 3 \cos t - 6 \sin t.$$

Therefore

$$\mathbf{r}(t) = \langle 3 \cos t, 3 \sin t, 3 \cos t - 6 \sin t \rangle, \quad 0 \leq t < 2\pi.$$

Step 3: Initial conditions At $t = 0$,

$$\mathbf{r}(0) = \langle 3, 0, 3 \rangle.$$

For small positive t , $\sin t > 0$, so $y = 3 \sin t > 0$; the required initial direction is satisfied.

Verification. Squaring the first two components gives $9(\cos^2 t + \sin^2 t) = 9$, and the third component is exactly the first minus twice the second. The half-open interval avoids repeating the starting point.