

Question 4

Two moving objects have position functions

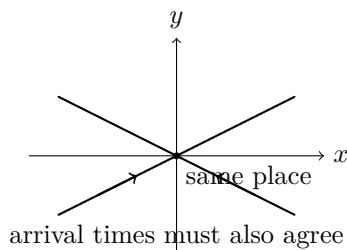
$$\mathbf{r}_1(t) = \langle t, t^2, 1 - t \rangle, \quad \mathbf{r}_2(s) = \langle 2 - s, s, s - 1 \rangle.$$

Tasks

1. Determine whether their geometric paths intersect.
2. If they do, find the parameter value on each path and the intersection point.
3. If both parameters represent the same clock time, decide whether the objects collide.

Question 4 – Solution

Strategy. A path intersection permits different parameters t and s . A collision additionally requires the same time.



Step 1: Match coordinates Equality of the two position vectors requires

$$t = 2 - s, \quad t^2 = s, \quad 1 - t = s - 1.$$

The first and third equations both simplify to

$$t + s = 2.$$

Using $s = t^2$ gives

$$t + t^2 = 2 \implies t^2 + t - 2 = 0 \implies (t - 1)(t + 2) = 0.$$

Step 2: Check both candidates If $t = 1$, then $s = t^2 = 1$, and all equations hold. If $t = -2$, then $s = 4$, but $t + s = 2$ also holds. Thus there are two path intersections:

$$\boxed{\langle 1, 1, 0 \rangle \quad (t, s) = (1, 1)}, \quad \boxed{\langle -2, 4, 3 \rangle \quad (t, s) = (-2, 4)}.$$

Step 3: Collision test With a common clock, a collision requires $t = s$. Only $(t, s) = (1, 1)$ satisfies that requirement. Therefore the objects collide at

$$\boxed{t = 1 \text{ at } \langle 1, 1, 0 \rangle}.$$

Verification. Direct substitution into both vector functions reproduces each path-intersection point.