

Question 3

A particle follows

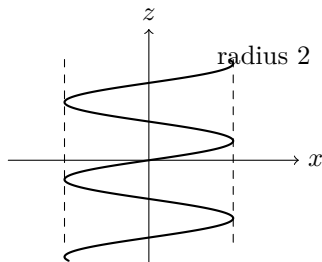
$$\mathbf{r}(t) = \langle 2 \cos t, 2 \sin t, t/\pi \rangle, \quad 0 \leq t \leq 4\pi.$$

Without calculus:

1. Describe the trace geometrically and state its axis and radius.
2. Find every point where the particle meets the plane $z = 1$.
3. Determine the vertical rise during one complete revolution and the total number of revolutions.

Question 3 – Solution

Strategy. Use the identity $\cos^2 t + \sin^2 t = 1$ and compare the angular parameter with the height.



Step 1: Cylindrical constraint

$$x^2 + y^2 = 4 \cos^2 t + 4 \sin^2 t = 4.$$

Thus the trace lies on the cylinder of radius 2 about the z -axis. Since $z = t/\pi$ steadily increases while (x, y) rotates counterclockwise, the curve is a circular helix.

Step 2: Plane intersection Setting $z = 1$ gives

$$\frac{t}{\pi} = 1 \implies t = \pi.$$

This value is in the interval, and therefore the only point is

$$\mathbf{r}(\pi) = \langle -2, 0, 1 \rangle.$$

Step 3: Pitch and turns One revolution changes t by 2π , so

$$\Delta z = \frac{t + 2\pi}{\pi} - \frac{t}{\pi} = 2.$$

The interval length 4π contains $4\pi/(2\pi) = 2$ revolutions. Hence the rise per turn is $\boxed{2}$ and the curve makes $\boxed{2}$ complete turns.