

Question 2

Consider

$$\mathbf{r}(t) = \langle 2t - 1, t^2 + 1, 3t + 4 \rangle, \quad -2 \leq t \leq 2.$$

Tasks

1. Eliminate t and give two Cartesian equations satisfied by the trace.
2. Identify the curve and its containing plane.
3. Find its two endpoints and indicate the direction of travel.

Question 2 – Solution

Strategy. Solve the simplest component for the parameter and substitute it into the other two components.

Step 1: Recover the parameter Since $x = 2t - 1$,

$$t = \frac{x+1}{2}.$$

Step 2: Eliminate t Substitution into $y = t^2 + 1$ gives

$$y = \frac{(x+1)^2}{4} + 1.$$

Also,

$$z = 3\left(\frac{x+1}{2}\right) + 4 = \frac{3}{2}x + \frac{11}{2},$$

or $3x - 2z + 11 = 0$. Hence the trace is the part of the parabola

$$y = \frac{(x+1)^2}{4} + 1, \quad 3x - 2z + 11 = 0$$

for which $-5 \leq x \leq 3$. It lies in the stated plane.

Step 3: Endpoints and orientation

$$\mathbf{r}(-2) = \langle -5, 5, -2 \rangle, \quad \mathbf{r}(2) = \langle 3, 5, 10 \rangle.$$

As t increases, $x = 2t - 1$ increases, so the curve is traversed from the first point to the second.

Verification. Substituting either endpoint into both Cartesian equations confirms both constraints exactly.