

Question 1

Let

$$\mathbf{r}(t) = \left\langle \sqrt{t+2}, \ln(5-t), \frac{1}{t^2-1} \right\rangle.$$

Tasks

1. Determine the exact domain of $\mathbf{r}$.
2. Evaluate $\mathbf{r}(2)$.
3. Explain what fails at every excluded endpoint or isolated value.

Question 1 – Solution

Strategy. A vector function is defined only where all three component functions are defined. We therefore intersect their domains.

Step 1: Square-root condition Real values require

$$t + 2 \geq 0 \implies t \geq -2.$$

Step 2: Logarithm condition The logarithm requires a strictly positive argument:

$$5 - t > 0 \implies t < 5.$$

Step 3: Denominator condition We must have

$$t^2 - 1 \neq 0 \implies t \neq -1, 1.$$

Intersecting the three conditions gives

$$\boxed{\text{Dom}(\mathbf{r}) = [-2, -1) \cup (-1, 1) \cup (1, 5]}.$$

Step 4: Evaluation

$$\boxed{\mathbf{r}(2) = \left\langle 2, \ln 3, \frac{1}{3} \right\rangle}.$$

Verification. At $t = -2$ the square root is zero, so that endpoint is included. At $t = 5$ the logarithm sees zero; below -2 its first component is nonreal; and at $t = \pm 1$ the third component divides by zero.