

Question 9

Let

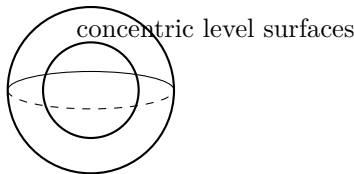
$$F(x, y, z) = \ln(4 - x^2 - y^2 - z^2).$$

Tasks

1. Find its domain and range.
2. Describe every level surface $F = c$.
3. Determine how the level radius changes as c increases.

Question 9 – Solution

Strategy Enforce positivity of the logarithm's argument and solve $F = c$ by exponentiating.



Step 1: Domain We need

$$4 - x^2 - y^2 - z^2 > 0,$$

so the domain is the open ball $\boxed{x^2 + y^2 + z^2 < 4}$.

Step 2: Range The logarithm's argument takes every value in $(0, 4]$. Therefore F takes every value in

$$\boxed{(-\infty, \ln 4]}.$$

The maximum $\ln 4$ occurs at the origin; the function tends to $-\infty$ near the boundary.

Step 3: Levels From $F = c$,

$$4 - r^2 = e^c \implies \boxed{r = \sqrt{4 - e^c}}, \quad c \leq \ln 4.$$

These are concentric spheres, degenerating to the origin at $c = \ln 4$. As c increases, e^c increases and the radius decreases.