

**Question 7**

A quadratic response has the form

$$q(x, y) = ax^2 + bxy + cy^2 + d.$$

It satisfies  $q(1, 0) = 6$ ,  $q(0, 1) = 4$ ,  $q(1, 1) = 15$ , and  $q(0, 0) = 1$ .

**Tasks**

1. Determine  $q$ .
2. Compute  $q(2, -1)$ .
3. Classify its level curves after removing the cross term by factoring.

**Question 7 – Solution**

**Strategy** Substitute the four samples one at a time.

**Step 1: Coefficients** From  $q(0, 0) = 1$ ,  $d = 1$ . Then

$$a + d = 6 \implies a = 5, \quad c + d = 4 \implies c = 3.$$

Finally,

$$a + b + c + d = 15 \implies 5 + b + 3 + 1 = 15 \implies b = 6.$$

Thus  $\boxed{q(x, y) = 5x^2 + 6xy + 3y^2 + 1}$ .

**Step 2: Evaluate**  $q(2, -1) = 20 - 12 + 3 + 1 = \boxed{12}$ .

**Step 3: Geometry** Complete a square:

$$5x^2 + 6xy + 3y^2 = 5 \left( x + \frac{3}{5}y \right)^2 + \frac{6}{5}y^2.$$

Both coefficients are positive, so  $q = k > 1$  gives rotated ellipses,  $q = 1$  gives only the origin, and  $q < 1$  gives no real points.