

Question 4

A temperature field is

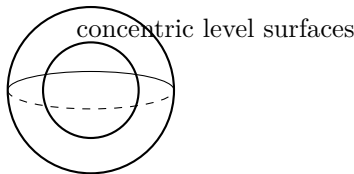
$$T(x, y, z) = 100e^{-(x^2+y^2+z^2)/4}.$$

Tasks

1. Describe the level surface $T = T_0$ for $0 < T_0 \leq 100$.
2. Find the 50-degree level radius.
3. Along $(t, t, 0)$, find when the temperature first reaches 50.

Question 4 – Solution

Strategy Take natural logarithms after isolating the exponential.



Step 1: General level From $T_0/100 = e^{-r^2/4}$,

$$\ln(T_0/100) = -r^2/4 \implies \boxed{r^2 = 4 \ln(100/T_0)}.$$

Thus levels are concentric spheres; $T_0 = 100$ degenerates to the origin.

Step 2: Fifty-degree surface For $T_0 = 50$, $r^2 = 4 \ln 2$, so $\boxed{r = 2\sqrt{\ln 2}}$.

Step 3: Path On $(t, t, 0)$, $r^2 = 2t^2$. Hence

$$2t^2 = 4 \ln 2 \implies t = \pm \sqrt{2 \ln 2}.$$

Starting at $t = 0$ and moving with increasing t , the first time is $\boxed{t = \sqrt{2 \ln 2}}$.