

Question 3

For positive inputs define

$$F(x, y) = \frac{xy}{x + y}.$$

Tasks

1. Show how F changes when both inputs are scaled by $s > 0$.
2. Derive the level curve $F = c > 0$.
3. State its domain branches and asymptotes.

Question 3 – Solution

Strategy Substitute scaled inputs, then solve the level equation explicitly for y .

Step 1: Scaling Direct substitution gives

$$F(sx, sy) = \frac{s^2 xy}{s(x+y)} = sF(x, y).$$

Thus the output has degree-one scaling.

Step 2: Level curve Set $xy/(x+y) = c$:

$$xy = cx + cy \implies y(x - c) = cx \implies \boxed{y = \frac{cx}{x - c}}.$$

For positive x, y , the denominator must be positive, so $x > c$; symmetry also implies $y > c$.

Step 3: Geometry Rewrite $y = c + c^2/(x - c)$. The asymptotes are $\boxed{x = c}$ and $\boxed{y = c}$, and the positive-input level is the branch with $x > c, y > c$. Substitution back into F returns c .