

Question 2

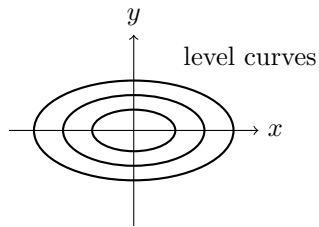
For $f(x, y) = x^2 + 4y^2$, analyze the level curves $f(x, y) = c$.

Tasks

1. Classify them for $c < 0$, $c = 0$, and $c > 0$.
2. Find semiaxes and enclosed area for $c > 0$.
3. Explain how the contours determine the graph's shape.

Question 2 – Solution

Strategy Treat c by sign and normalize the positive-level equation.



Step 1: Cases Because squares are nonnegative, $c < 0$ gives no points. For $c = 0$, both squares vanish, giving $(0, 0)$. For $c > 0$,

$$\frac{x^2}{c} + \frac{y^2}{c/4} = 1.$$

Thus the semiaxes are $\sqrt{c}$ and $\sqrt{c}/2$.

Step 2: Area The enclosed area is

$$A(c) = \pi (\sqrt{c}) \left(\frac{\sqrt{c}}{2} \right) = \boxed{\frac{\pi c}{2}}.$$

Step 3: Graph Higher levels produce nested, expanding ellipses centered at the origin. Since $z = x^2 + 4y^2 \geq 0$ and increases quadratically, the graph is an upward elliptic paraboloid, steeper in the y -direction.