

Question 9

Classify the real point sets

$$x^2 + y^2 + z^2 - 2xy - 2xz + 2yz = c$$

Tasks

1. Factor the quadratic expression.
2. Classify the real point set for $c < 0$, $c = 0$, and $c > 0$.
3. Explain the degeneration geometrically.

Question 9 – Solution

Strategy Test whether the quadratic expression is a perfect square.

Step 1: Factor Expanding

$$(x - y - z)^2 = x^2 + y^2 + z^2 - 2xy - 2xz + 2yz.$$

Thus the equation is simply

$$(x - y - z)^2 = c.$$

Case analysis If $c < 0$, no real square can equal c , so the set is empty. If $c = 0$, then $x - y - z = 0$, a single plane. If $c > 0$, taking square roots gives

$$x - y - z = \pm\sqrt{c},$$

so the set is two distinct parallel planes.

Conclusion The classification is $\boxed{\emptyset}$, $\boxed{\text{one plane}}$, and $\boxed{\text{two parallel planes}}$ for the three cases. Substitution into the factored form verifies each result.